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INDECOMPOSABLE SYZYGY-MODULES OF HIGH RANK OVER HYPERSURFACE RINGS

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In this paper we construct certain indecomposable syzygy modules over graded hypersurface rings and give a lower bound of their rank.

Thereby we show that there is no upper bound for the ranks of the indecomposable maximal Cohen–Macaulay modules over a graded hypersurface ring of dimension at most two and of multiplicity at most three.

Let $R=\bigoplus_{i\geq 0} R_i$ be a graded ring, where $R_0=k$ is a field and $R=k[R_1]$ is finitely generated over k. We denote by $\mathfrak{m}$ the irrelevant maximal ideal of R, by h_R the Hilbertfunction of R and by $\Omega^i_R(M)$ the ith syzygy-module of a graded R-module M.

Theorem. *Let $R=k[X_0,\ldots,X_d]/f$ be a hypersurface ring, where f is a homogeneous polynomial of degree $e\geq 3$. Then for any integer $a\geq e$ the Cohen–Macaulay module $\Omega^{d+1}_R(R/\mathfrak{m}^a)$ is indecomposable of rank $\geq h_R(a-1)$.*

For the proof of this theorem we will use an indecomposability criterion for Cohen–Macaulay modules.

Therefore we need some more notations. We consider the category of finitely generated graded R-modules with the homogeneous homomorphisms of degree 0 as morphisms. By $\mathcal{M}(R)$ we will denote the full subcategory of the maximal Cohen–Macaulay modules.

We call a module M *equigenerated* if all elements of a minimal system of homogeneous generators of M have the same degree.

Lemma 1 (indecomposability criterion). *Suppose that $M\in\mathcal{M}(R)$ has no free direct summand, and that there exists a chain of submodules $(0)\subseteq M_0\subseteq\cdots\subseteq M_t=M$ such that the following conditions hold:*

(1) *The factors M_i/M_{i-1} are equigenerated of degree a_i and $a_0<a_1<\cdots<a_t$.*
(2) *The factors M_i/M_{i-1} are indecomposable for $1\leq i\leq t$ and M_0 is free.*
(3) *The natural inclusions $M_i/M_{i-1}\to M/M_{i-1}$ do not split.*

Then M is indecomposable.

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If k is algebraically closed of char $\neq 2$ and the graded ring R is an isolated hypersurface singularity of dimension ≥ 2, it follows from works of Knörrer [8] and of Buchweitz, Greuel and Schreyer [2], that R is of finite Cohen–Macaulay representation type if and only if the multiplicity of R is at most two. Combining the theorem with [1] we obtain

Corollary. *Suppose R is a graded Gorenstein domain of dimension ≥ 2 with isolated singularity and multiplicity $e(R)$. Then the following conditions are equivalent*:

(a) $e(R) = \operatorname{codim} R + 1$;

(b) *R is of bounded rank Cohen–Macaulay representation type.* □

The condition under (b) means that there exists an upper bound for the ranks of all indecomposable modules of $\mathcal{M}(R)$.

At the same time Dieterich [3] and Yoshino [9] have independently proved results which imply: If R is an isolated hypersurface singularity, then R is of finite Cohen–Macaulay representation type if and only if R is of bounded rank Cohen–Macaulay representation type.

The case $\dim R = 1$ (R reduced, char $R = 0$) was treated by Greuel and Knörrer in [5]. Their results imply: If R is of bounded rank Cohen–Macaulay representation type, then R is a simple plane curve singularity or $e(R) = \operatorname{codim} R + 1$.

Kiyek and Steinke [7] showed that the assumption char $R = 0$ is unnecessary. Notice that for the implication (b)$\Rightarrow$(a) of the corollary one definitely needs $\dim R \geq 2$. In fact, the plane curve singularity $R = \mathbb{C}[X, Y]/(X^2Y + Y^3)$ is simple and therefore even of finite Cohen–Macaulay representation type, but $e(R) = 3 > 2 = \operatorname{codim} R + 1$.

The corollary follows quite easily from the theorem: Observe first that both conditions in the corollary imply that R is a hypersurface ring. This is obvious for condition (a). To see that R is a hypersurface ring if (b) holds we use a similar argument as in [6].

It amounts to show that the Bettinumbers $\beta_i(k) := \dim_k \operatorname{Tor}_i^R(k, k)$ are bounded. For $i \geq d$ let $\Omega^i(k) = \bigoplus_{j=1}^{t_i} M_{ij}$ be the decomposition into indecomposable maximal Cohen–Macaulay modules. Since $\Omega^i(N)$ of an indecomposable maximal Cohen–Macaulay module N is again indecomposable, it follows that $t_{i+1} \leq t_i$ for all $i \geq d$.

Consequently, if n is the upper bound of ranks of indecomposable maximal Cohen–Macaulay modules, then rank $\Omega^i(k) \leq t_d \cdot n$ for all $i \geq d$. It follows that $\beta_i(k) \leq 2 \cdot t_d \cdot n$ for all $i \geq d$.

Now (a)$\Rightarrow$(b) follows from [1], while (b)$\Rightarrow$(a) is a consequence of the theorem. In fact, if $e := e(R) > \operatorname{codim} R + 1$, then $e \geq 3$ since R is a hypersurface ring. Hence by the theorem there exist indecomposable maximal Cohen–Macaulay modules of rank $\geq h_R(a-1)$ for all $a \geq e$, and since $\dim R \geq 2$ the Hilbert-function h_R is strictly increasing.

Proof of Lemma 1. We will show by induction on $t-i$ that M/M_i is indecomposable for $-1 \leq i \leq t-1$, where $M_{-1} := 0$.

For $i = t-1$ the assertion follows from (2).

Assume that we have shown the assertion for some $0 \leq i \leq t-1$. We have the short exact sequences

$$0 \to M_i/M_{i-1} \to M/M_{i-1} \to M/M_i \to 0 .$$

The least degree of the generators of M/M_{i-1} is a_i, and M_i/M_{i-1} is the submodule generated by the elements of degree a_i in M/M_{i-1}. Suppose $M/M_{i-1} = B \oplus C$, where $B \neq 0 \neq C$. Then $(M_i/M_{i-1})_{a_i} = (M/M_{i-1})_{a_i} = B_{a_i} \oplus C_{a_i}$. Let $V := \langle B_{a_i} \rangle$ be the submodule of B generated by the elements of B_{a_i} and $W = \langle C_{a_i} \rangle \subseteq C$, then $V \cap W = 0$, and since $(M_i/M_{i-1})_{a_i} = B_{a_i} \oplus C_{a_i}$ it follows that $M_i/M_{i-1} = V + W$, so that $M_i/M_{i-1} = V \oplus W$.

If $i \geq 1$ we may assume that $M_i/M_{i-1} = V \subseteq B$ since M_i/M_{i-1} is indecomposable by (2). It follows that $M/M_i = B/V \oplus C$ and since $C \neq 0$ we get $B/V = 0$ because by induction hypothesis M/M_i is indecomposable.

Now $B = V = M_i/M_{i-1}$ so that the inclusion $M_i/M_{i-1} \to M/M_{i-1}$ splits. This contradicts (3).

If $i = 0$ we consider the short exact sequence $0 \to M_0 \to M \to M/M_0 \to 0$, where M_0 is free and $M/M_0 = B/V \oplus C/W$. Since M/M_0 is indecomposable by induction hypothesis, we may assume that $B = V$. But $V \oplus W = M_0$, so that B is a direct summand of the free module M_0 and hence B is free, a contradiction to the assumption that M has no free direct summand. □

The next lemmata serve the purpose to define a suitable filtration on $\Omega^{d+1}(R/\mathfrak{m}^a)$ and to verify that the conditions of Lemma 1 are satisfied for this filtration.

Now let $A = k[X_0, \ldots, X_d]$, and let $R = A/f$ be a graded hypersurface ring of multiplicity e. Let I be the ideal $(f, \mathfrak{n}_A^a)$ in A, where $\mathfrak{n}_A$ denotes the irrelevant maximal ideal of A and where $a \geq e$.

Lemma 2. *$\Omega_A^1(I)$ is equigenerated of degree $a+1$ and has a linear resolution.*

Proof. We proceed by induction on d.

If $d = 0$, then $\Omega_A^1(I) = 0$.

Let $d > 0$. We may assume that X_d is a nonzerodivisor on R. For an R-module M we set $\bar{M} := M/X_dM$ and for $g \in A$ we set $\bar{g} := g + (X_d)$. There exists a short exact sequence $0 \to \Omega_A^1(I) \to F \to I \to 0$, where F is free. This sequence gives rise to the exact sequence

$$0 \to \overline{\Omega_A^1(I)} \to \bar{F} \to \bar{I} \to 0 . \tag{1}$$

The exact sequence $0 \to I \to A \to A/I \to 0$ yields the exact sequence

$$0 \to \operatorname{Tor}_1^A(\bar{A}, A/I) \to \bar{I} \to \bar{A} \to \overline{A/I} \to 0 .$$

Let $J := (\bar{f}, \mathfrak{n}_{\bar{A}}^a)$, then $\overline{A/I} = \bar{A}/J$ and hence we obtain the exact sequence

$$0 \to \operatorname{Tor}_1^A(\bar{A}, A/I) \to \bar{I} \to J \to 0 . \tag{2}$$

To compute $\operatorname{Tor}_1^A(\bar{A}, A/I)$ we tensorize the exact sequence $0 \to A(-1) \xrightarrow{X_d} A \to \bar{A} \to 0$ with $A/I \simeq R/\mathfrak{m}^a$ and find that

$$\begin{aligned}\operatorname{Tor}_1^A(\bar{A}, A/I) &\simeq \operatorname{Ker}(R/\mathfrak{m}^a(-1) \xrightarrow{X_d} R/\mathfrak{m}^a) \simeq (\mathfrak{m}^{a-1}/\mathfrak{m}^a)(-1) \\ &\simeq k^{h_R(a-1)}(-a) .\end{aligned}$$

Using (2) we get the exact sequence

$$0 \to k^{h_R(a-1)}(-1) \to \bar{I} \to J \to 0 . \tag{3}$$

We claim that this sequence splits. For this it suffices to show that $\mu(I) = \mu(\bar{I}) = h_R(a-1) + \mu(J)$. We have $\mu(I) = 1 + h_R(a)$ and $\mu(J) = 1 + h_{\bar{R}}(a)$. Since $0 \to R(-1) \xrightarrow{X_d} R \to \bar{R} \to 0$ is exact, it follows that $h_R(a) = h_{\bar{R}}(a) + h_R(a-1)$, which implies the above equality.

Now (3) implies that $\bar{I} \simeq k^{h_R(a-1)}(-a) \oplus J$. Clearly $\Omega_A^1(k^{h_R(a-1)}(-a))$ is equigenerated of degree $a+1$ and has a linear resolution. Using the induction hypothesis it follows that $\Omega_A^1(I)$ is equigenerated of degree $a+1$ and has a linear resolution. These properties behave well under deformation, and hence the lemma is proved. □

Let $(F_., \varphi_.) : 0 \to F_{d+1} \xrightarrow{\varphi_{d+1}} F_d \to \cdots \to F_1 \xrightarrow{h} F_0 \to 0$ be the minimal homogeneous A-resolution of $R/\mathfrak{m}^a \simeq A/I$. Then rank $F_{d+1} = r(R/\mathfrak{m}^a) = h_R(a-1)$, where r denotes the Cohen–Macaulay type of $R/\mathfrak{m}^a$.

Further let $(G_., \psi_.)$ be the complex $F_. \otimes_A R$ and $U_i := B_{i+1}(G_.) = \operatorname{Im} \psi_{i+1}$ the boundaries of $G_.$ for $i \geq 0$. Then $U_0 = \mathfrak{m}^a$ and for $i > 0$ the complex

$$0 \to G_{d+1} \xrightarrow{\psi_{d+1}} G_d \longrightarrow \cdots \xrightarrow{\psi_{i+2}} G_{i+1} \to U_i \to 0$$

is exact and linear by Lemma 2.

Lemma 3. *U_i is indecomposable for $0 \leq i \leq d-1$, and $U_d \simeq G_{d+1}$.*

Proof. For $i = 0$ the assertion is clear. Hence we only have to deal with $i > 0$.

We first verify that none of the U_i has a free direct summand for $1 \leq i \leq d-1$. In fact, suppose that U_i has a free direct summand, then after a suitable choice of the basis of the G_{i+1} the matrix $\mathfrak{A}_{i+2}$ of ψ_{i+2} has a zero column. Let $\mathfrak{B}_{i+2}$ be the matrix describing φ_{i+2}, then $\mathfrak{A}_{i+2}$ is obtained from $\mathfrak{B}_{i+2}$ by reduction modulo f. Since $\mathfrak{B}_{i+2}$ is a matrix of linear forms, which has a zero column after reduction mudulo f, the corresponding column of $\mathfrak{B}_{i+2}$ is already zero. But this means that $V_i := B_{i+1}(F_.)$ has a free direct summand as well.

Because the exact sequence $0 \to V_i \to F_i \to V_{i-1} \to 0$ is minimal, $V_i \subseteq \mathfrak{m} F_i$ and therefore $\beta(V_i) \subseteq \mathfrak{m}$ for all $\beta \in F_i^* := \mathrm{Hom}_A(F_i, A)$. The dual sequence $0 \to V_{i-1}^* \to F_i^* \to V_i^* \to 0$ stays exact since $\mathrm{Ext}_A^1(V_{i-1}, A) \simeq \mathrm{Ext}_A^{i+1}(A/I, A) = 0$ for $i < d$.

Now we conclude that $\beta(V_i) \subseteq \mathfrak{m}$ for all $\beta \in V_i^*$ and consequently V_i has no free direct summand; a contradiction.

We now assume $U_i = B \oplus C$, where $B \neq 0 \neq C$ are not free. Then $U_{i+1} \simeq \Omega_A^1(B) \oplus \Omega_A^1(C)$, where $\Omega_A^1(B) \neq 0 \neq \Omega_A^1(C)$, so that U_{i+1} is decomposable as well. By induction it follows that U_{d-1} decomposes, say $U_{d-1} = D \oplus E$, where $D \neq 0 \neq E$.

Since $\omega_{R/\mathfrak{m}^a} \simeq \mathrm{Coker}\, \psi_{d+1}^* \simeq \mathrm{Ext}_R^1(U_{d-1}, R) \simeq \mathrm{Ext}_R^1(D, R) \oplus \mathrm{Ext}_R^1(E, R)$ is indecomposable, we may assume that $\mathrm{Ext}_R^1(D, R) = 0$, so that $D \in \mathcal{M}(R)$, since clearly $\mathrm{Ext}_R^i(D, R) = 0$ for $i > 1$. But $pd_R U_{d-1} = 1$, hence $pd_R D \leq 1$ and therefore D must be free; a contradiction. □

Applying Lemma 1 to $\Omega^{d+1}(R/\mathfrak{m}^a)$ we need to know that $\Omega^{d+1}(R/\mathfrak{m}^a)$ has no free direct summand. But this is clear since $\Omega^{d+1}(R/\mathfrak{m}^a) = \Omega^1(\Omega^d(R/\mathfrak{m}^a))$, and $\Omega^d(R/\mathfrak{m}^a)$ is already Cohen–Macaulay.

For a better understanding of the structure of $\Omega^{d+1}(R/\mathfrak{m}^a)$ we now describe a (minimal) R-resolution of $R/\mathfrak{m}^a$, following the description of Eisenbud [4]:

For $j \geq 0$ let $G_i[-j] := F_{i-2j} \otimes_A R(-je)$ and define the graded R-module $H_.$ by $H_i := \bigoplus_{j \geq 0} G_i[-j]$. $H_.$ can be made an R-resolution $(H, \partial_.)$ of $R/\mathfrak{m}^a$. The differential $\partial_i : H_i \to H_{i-1}$ results from a family of homogeneous homomorphisms $(s_\alpha)_{\alpha \geq 0}$, $s_\alpha : F_. \to F_{.+2\alpha-1}$. These homomorphisms induce homogeneous homomorphisms

$$s_\alpha(j) : G_.[-j] \to G_{.-1}[-(j-\alpha)]$$

with the following properties:

(1) $s_\alpha(j) = 0$ for $j < \alpha$;

(2) $s_\alpha(j+1) = s_\alpha(j)$ for $j \geq \alpha$;

(In this sense s_α does not depend upon j and therefore we sometimes simply write s_α again.)

(3) s_0 is the differential $\psi_.$ of $G_.$;

(4) If $\gamma \geq 1$, then $\sum_{\alpha+\beta=\gamma} s_\alpha s_\alpha = 0$;

(5) $s_\alpha = 0$, if $\alpha > \frac{1}{2}(d+1)$.

Now $\partial_i : H_i \to H_{i-1}$ is defined by $\partial_i|_{G_i[-j]} := \bigoplus_{\alpha \geq 0} s_\alpha(j)$. After a suitable choice of the bases of H_i and H_{i-1} we can identify the map ∂_i with the matrix $(s_i(j))_{i,j}$, where the number of the rows resp. the columns are given by the number of direct summands $\neq 0$ of H_i resp. H_{i-1} (of the form $G_i[-j]$ resp. $G_{i-1}[-j]$).

$$(s_i(j))_{i,j} = \begin{bmatrix} s_0(0) & s_1(1) & s_2(2) & \cdots \\ & s_0(1) & s_1(2) & \cdots \\ & & s_0(2) & \cdots \\ & & & \ddots \\ 0 & & & \end{bmatrix}.$$

Next we are going to modify $(H_\bullet, \partial_\bullet)$ to obtain a minimal R-resolution of $R/\mathfrak{m}^a$.

For all $i > 0$ we have $G_{2i}[-i] \simeq R(-ie)$ and $G_{2i-1}[-(i-1)] \simeq R^{\beta_1}(-(i-1)e - a) \oplus R(-ie)$, where $\beta_1 = h_R(a)$. The differential ∂_i maps the generator of $G_{2i}[-i]$ onto the generator of $R(-ie)$ in $G_{2i-1}[-(i-1)]$. Therefore we may cancel these two copies of $R(-ie)$ in $H_\bullet$ for all $i > 0$. We again denote the resulting quotient complex by $(H_\bullet, \partial_\bullet)$.

Lemma 4. *If $e \geq 2$, then $(H_\bullet, \partial_\bullet)$ is a minimal R-resolution of $R/\mathfrak{m}^a$.*

Proof. We compute the degrees of the entries of the maps $s_\alpha(j)$: $G_i[-j] \to G_{i-1}[-(j-\alpha)]$ with respect to a homogeneous basis of the corresponding modules.

By Lemma 2 the F_i are equigenerated of degree $i + a - 1$ for $i \geq 2$, and $F_1 \simeq A^{\beta_1}(-a) \oplus A(-e)$, so that after the above mentioned cancelling $G_i[-j]$ is equigenerated of degree $i - 2j + a - 1 + je$ and $G_{i-1}[-(j-\alpha)]$ is equigenerated of degree $(i-1) - 2(j-\alpha) + a - 1 + (j-\alpha)e$. It follows that the entries of $s_\alpha(j)$ have degree $\alpha(e-2) + 1 > 0$, since $\alpha \geq 2$ and $e \geq 2$. This yields the minimality of $(H_\bullet, \partial_\bullet)$ at ∂_j for $j > 1$. The minimality at ∂_1 is obvious. $\square$

Now we define the filtration on $\Omega^{d+1}(R/\mathfrak{m}^a)$ that is needed in Lemma 1.

We first define a filtration on H. Let

$$\mathcal{F}_q H_i := \bigoplus_{q \geq j \geq 0} G_i[-j]$$

and

$$\mathcal{F}_q \partial_i := \partial_i|_{\mathcal{F}_q H_i} = (s_i(j))_{0 \leq i, j \leq q}\,.$$

Then $\mathcal{F}_{-1} H_\bullet = 0$, $\mathcal{F}_q H_\bullet$ is a subcomplex of $\mathcal{F}_{q+1} H_\bullet$, so that

$$0 \subseteq \mathcal{F}_0 H_\bullet \subseteq \cdots \subseteq \mathcal{F}_q H_\bullet \subseteq \mathcal{F}_{q+1} H_\bullet \subseteq \cdots$$

If we further let $H_i[-p] := \bigoplus_{j \geq p} G_i[-j]$ we get

Lemma 5. *There are two natural short exact sequences of complexes*

(a) $\qquad 0 \to \mathcal{F}_{q-1} H_\bullet \to H_\bullet \to H_\bullet[-q] \to 0$

and

(b) $$0 \to \mathscr{F}_{q-1}H. \to \mathscr{F}_q H. \to G.[-q] \to 0 . \qquad \square$$

The proof is clear by the definitions of these complexes.

To simplify the notation we now set $M = \Omega^{d+1}(R/\mathfrak{m}^a)$. The filtration on $H.$ induces a filtration on $M \simeq B_{d+1}(H.)$, i.e. $M_q := B_{d+1}(\mathscr{F}_q H.)$, so that $M_{-1} = 0$, $M_0 \simeq G_{d+1}$ and $M_q = M$ for $q \gg 0$.

Lemma 6. *If $M_{q-1} \neq M$, the following sequences are exact:*

(a) $$0 \to M_{q-1} \to M \to (B_{d+1-2q}(H.))(-qe) \to 0$$

and

(b) $$0 \to M_{q-1} \to M_q \to U_{d-2q}(-qe) \to 0 .$$

Proof (a) The sequence (a) of Lemma 5 gives rise to the exact sequence $0 \to B_{d+1}(\mathscr{F}_{q-1}H.) \to B_{d+1}(H.) \to B_{d+1}(H.[-q]) \to 0$, so that the assertion follows since $B_{d+1}(H.[-q]) \simeq B_{d+1-2q}(H.)(-qe)$ if $d+1-2q > 0$. But this inequality is clear since $M_{q-1} \neq M$ implies that $q \leq \frac{1}{2}d$.

(b) The sequence (b) of Lemma 5 implies the exact sequence

$$0 \to B_{d+1}(\mathscr{F}_{q-1}H.) \to B_{d+1}(\mathscr{F}_q H.) \to B_{d+1}(G.[-q]) \to 0 .$$

Again using $M_{q-1} \neq M$ it follows that

$$B_{d+1}(G.[-q]) \simeq B_{d+1-2q}(G.)(-qe) \simeq U_{d-2q}(-qe) . \qquad \square$$

Lemma 7. *The natural inclusion $M_q/M_{q-1} \to M/M_{q-1}$ does not split.*

Proof. Suppose that the natural inclusion $M_q/M_{q-1} \to M/M_{q-1}$ splits. Then by Lemma 6 U_{d-2q} is a direct summand of $B_{d+1-2q}(H.)$, so that $\Omega^i(U_{d-2q})$ is a direct summand of $\Omega^i(B_{d+1-2q}(H.))$ for $i \geq 0$. For $i = 2q$ this implies that G_{d+1} is a direct summand of M; a contradiction. $\square$

Now we are ready for the proof of the theorem. The module M and the chain of submodules $(M_q)_{q \geq -1}$ satisfy all conditions of Lemma 1. The factors $M_q/M_{q-1} \simeq U_{d-2q}(-qe)$ (Lemma 6) are equigenerated of degree $a_q = d - 2q + a + qe$. Since $a_{q+1} = d - 2(q+1) + a + (q+1)e$ we see that $a_{q+1} > a_q$ if and only if $e > 2$.

Further by Lemma 3 the factors M_q/M_{q-1} are indecomposable for $q > 0$ and $M_0 \simeq G_{d+1}$ is free. Finally the natural inclusion $M_q/M_{q-1} \to M/M_{q-1}$ does not split by Lemma 7, so that M is indecomposable.

Notice that $G_{d+1} \subseteq M$ has rank $h_R(a-1)$. This implies rank $M \geq h_R(a-1)$, which completes the proof.

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